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$$f(x) = \frac{1}{(x+1)^2} + \frac{1}{(x+2)^2} + \cdots + \frac{1}{(x+n)^2} + f(x+n).$$

Letting n approach $+\infty$, we get, on account of (ii),

$$f(x) = \frac{1}{(x+1)^2} + \frac{1}{(x+2)^2} + \frac{1}{(x+3)^2} + \cdots \text{ to infinity.} \quad (2)$$

To show that (2) is a solution of (1), we observe that the series converges uniformly with respect to x over the right half plane including the axis of imaginaries, since $|x+n| \geq n$.

Hence,

$$\int_b^{b+1} f(x) dx = \left(\frac{1}{b+1} - \frac{1}{b+2} \right) + \left(\frac{1}{b+2} - \frac{1}{b+3} \right) + \cdots = \frac{1}{b+1},$$

for any path of integration lying in the above region.

The series (2) may be summed by means of the gamma function. Thus integrating between the limits from 0 to x , we have

$$\int_0^x f(x) dx = \left(\frac{1}{1} - \frac{1}{x+1} \right) + \left(\frac{1}{2} - \frac{1}{x+2} \right) + \cdots = S, \text{ say.}$$

Since S is uniformly convergent,

$$\begin{aligned} \int_0^x S dx &= \left\{ \frac{x}{1} - \log \left(1 + \frac{x}{1} \right) \right\} + \left\{ \frac{x}{2} - \log \left(1 + \frac{x}{2} \right) \right\} + \cdots \\ &= -\log \left[\left\{ e^{-(x/1)} \left(1 + \frac{x}{1} \right) \right\} \left\{ e^{-(x/2)} \left(1 + \frac{x}{2} \right) \right\} \cdots \right] \\ &= -\log \left(\frac{1}{\Gamma(x) e^{\gamma x}} \right) = \log \Gamma(x+1) + \gamma x. \end{aligned}$$

Hence,

$$f(x) = \frac{dS}{dx} = \frac{d^2}{dx^2} [\log \Gamma(x+1)].$$

The general solution of (1) subject to (i) but not to (ii) is now seen to be

$$f(x) = \frac{d^2}{dx^2} \log \Gamma(x+1) + \frac{d}{dx} \phi(x),$$

where $\phi(b+1) = \phi(b)$, and, therefore, $\phi(x)$ is any analytic function admitting the period 1.

Also solved by W. R. RANSOM and the PROPOSER.

414. Proposed by C. N. SCHMALL, New York City.

Among spherical triangles having the same base and equal altitudes, show that the isosceles triangle has the greatest vertical angle. Show that this is also true for plane triangles.

SOLUTION BY W. J. THOME, University of Detroit.

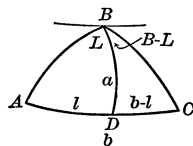
ABC is any spherical triangle. BD is a great circle arc perpendicular to AC . The angle B is broken up into two parts, L and $(B-L)$, and the corresponding opposite sides are l and $(b-l)$. Suppose B moves around on a small circle concentric with AC . Then BD remains constant in value.

In the $\triangle ABD$, $\cot L = \sin a \cot l$, and in the $\triangle BDC$, $\cot (B-L) = \sin a \cot (b-l)$. Now $B = L + (B-L) = \cot^{-1}(\sin a \cot l) + \cot^{-1}(\sin a \cot (b-l))$.

Differentiating,

$$\frac{dB}{dl} = \sin a \left[\frac{-\csc^2 l}{1 + [\sin a \cot l]^2} + \frac{\csc^2 (b-l)}{1 + [\sin a \cot (b-l)]^2} \right].$$

which is 0 if $b = 2l$. This evidently gives a maximum value of B . But if $b = 2l$, $AB = BC$ and the $\triangle$ is isosceles.



For the plane triangle, let ABC be any plane triangle. BD is a $\perp$ to AC from B . The vertex B may move anywhere on the straight line EF which is parallel to AC , thus keeping the altitude BD constant in value. The angle B is broken up into the two parts L and $(B - L)$ and the corresponding opposite sides are l and $(b - l)$.

In the $\triangle ABD$, $\tan L = l/a$ and in the $\triangle BDC$,

$$\tan (B - L) = \frac{b - l}{a}.$$

Now

$$B = L + (B - L) = \tan^{-1} (l/a) + \tan^{-1} \frac{b - l}{a}.$$

Differentiating,

$$\frac{dB}{dl} = a \left[\frac{1}{a^2 + l^2} - \frac{1}{a^2 + (b - l)^2} \right]$$

which is 0 if $b = 2l$, giving a maximum value of B . But if $b = 2l$, $AB = BC$ and the triangle is isosceles.

Also solved by M. R. GAFFET.

MECHANICS.

326. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A uniform beam, of length $2l$, rests in equilibrium against a smooth vertical wall and upon a peg at a distance a from the wall, show that the inclination of the beam to the vertical is

$$\sin^{-1} \left(\frac{a}{l} \right)^{\frac{1}{3}}.$$

SOLUTION BY H. S. UHLER, Yale University.

In order to obtain the given inclination it is necessary to assume that the reaction of the peg is normal to the beam, for, if this be not the case, the beam may be in equilibrium on various slants depending upon the value of the coefficient of friction.

The necessary and sufficient condition for static equilibrium is that the lines of action of the three forces acting on the beam pass through a single point, I . These forces are obviously the weight of the beam, W , acting vertically downward, the horizontal reaction of the smooth wall, R , and the normal reaction of the peg, R' . $BC = CF = l$. $DF = a$. $P \equiv$ peg. Let $\overline{FP} = m$ and $\overline{FI} = n$. Since $\overline{IP}$ and $\overline{PD}$ are, respectively, the perpendiculars dropped from the vertices of the right angles upon the hypotenuses of the triangles CIF and FPI we have $n^2 = lm$ and $m^2 = an$, so that $a^3/m^3 = a/l$ or $a/m = (a/l)^{1/3}$. But $i = \sin^{-1} (a/m) = \sin^{-1} (a/l)^{1/3}$, as required.

Also solved by A. M. HARDING, L. A. WARREN, G. PAASWELL, PAUL CAPRON, O. S. ADAMS, and HORACE OLSON.

327. Proposed by C. N. SCHMALL, New York City.

An inclined plane makes an angle φ with the horizontal plane, and from its foot a body is projected upward at an angle ψ to the plane, and with a velocity v . Show that it will strike the plane perpendicularly if $\tan \psi = \frac{1}{2} \cot \varphi$ and that its range up the plane in this case will be

$$\frac{2v^2 \sin \varphi}{g(1 + 3 \sin^2 \varphi)}.$$